Model prediction for constant area, variable pressure drop in orifice plate characteristics in flow system

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ABSTRACT

The effect of density, pressure drop, viscosity and orifice area on the characteristics of fluid flow was examined in this paper. Also studied was the effect on the control pressure change of the constant area variable pressure drop meter as a proportional derivative control. The mathematical model developed to monitor and predict the control of the system is given as \( P - P_0 = 7.8/t - 0.06 + K_c + K_d \). The change in control pressure decreases with increase in proportional/derivative gain \((K_c, K_d)\) as well as increase in time. The Bernoulli’s principle was applied in describing the design principle, stability analysis and development of mathematical model of a pressure-based flow meter with a constant area, variable pressure drop; using an orifice plate with different fluid flowing through it. The developed formula relates pressure drop with the flow rate of a given fluid passing through the orifice. The formula obtained is then simulated using different fluids. In order to control the flow rate, of these fluid flowing through the model developed was related to a Proportional Derivative control (PD). Thereby getting knowledge on how the PD controller performs with respect to different fluids, with change in pressure, density and area of the pipe/orifice was presented in this paper. Finally information and results on the simulation and how the PD controller functional parameters of proportional gain and derivative gain influence the control system was examined in this research.

INTRODUCTION

Flow measurement is an important part of the chemical industry. It can be done in variety of ways. Both gas and liquid flows can be measured in volumetric and mass flow rate, example litres per second and kilogram per second. Instruments have been designed to aid in flow rate calculation. They have common name flow meter and various flow meters can be used depending on application and industry. These flow meters are classified according to the
principle surrounding their operation, and can thus be classified as follows: Mechanical flow meters, piston meter, rotating piston meter, variable area meter, turbine flow meter, woftmann meter, nutating disk meter, single jet meter, oval gear meter, pelton wheel and multiple jet meter. Pressure based meter—ventri meter, orifice plate, dali tube, pilot tube, multi-hole pressure probe meter; optical flow meters, thermal mass flow meters, electromagnetic flow meters, ultra sonic flow meters, coriols flow meters and laser droplet flow meter (Alan, 2001; Ang et al., 2005; Aris, 1994; Ayotamuno et al., 2006; Batchelor, 2002; King, 2010; Ogoni and Ukpaka, 2004; Roberson and Crowe, 1993; Scott, 2005; Wayne, 2006). In this paper orifice plate meter in terms of characteristics and effect of control system was studied specifically, when subjected into disturbance with fluid flow.

Process control is a means of controlling the quality of products in a given process. In order to use an instrument for optimum process control, the principles governing the instrument must be fully understood. It is therefore of great importance to study and understand these principles as it applies to controlling a process. The purpose of the study is to know the principle governing pressure based meters using a basis of the constant area, variable pressure drop meter in the measurement of the flow rates of different fluids. And also to know how this meter will work as a proportional derivative controller. The objectives of this study are as follows: understanding the design of the meter, developing and deriving a mathematical model of the meter based on its design, studying how various fluids are measured and the effects, and studying the meter as a proportional derivative controller.

The following are the contributions of this study to knowledge: good understanding of a pressure based meter, good understanding on the behaviour of an orifice meter as a proportional derivative controller (PD) and good understanding on the design principles of an orifice plate and pressure based meters. The scope of this paper covers, pressure based meters (constant area, variable pressure drop meters) using orifice plate. Investigation conducted by various research groups revealed that there are several types of flow meter that rely on Bernoulli’s principle, either by measuring the differential pressure within a constriction, or by measuring static and stagnation pressures to derive the dynamic pressure. These are: venturi meter, orifice plate, dali tube, pilot tube and multi-hole pressure probe (Ang et al., 2006; Bennett, 1993; Cunningham, 1951; Geankoplis, 1993; Liang, 2009; Minorsky, 1992; Yang, 2005). An orifice plate is a plate with a hole through it, placed in the flow; it constrains the flow, and measuring the pressure differential across the constriction gives the flow rate. It is basically a crude form of venturi meter, but with higher energy losses. There are three type of orifice: concentric, eccentric, and segmental.

Orifice plates are most commonly used for continuous measurement of fluid flow in pipes. They are also used in some small river systems to measure flow rates at locations where the river passes through a culvert or drain. Only a small number of rivers are appropriate for the use of the technology since the plate must remain completely immersed i.e the approach pipe must be full, and the river must be substantially free of debris.

In the natural environment large orifice plates are used to control onward flow in flood relief dams. In these structures a low dam is placed across a river and in normal operation the water flows through the orifice plate unimpeded as the orifice is substantially larger than the normal flow cross section. However, in floods, the flow rate rises and floods out the orifice plate which can then only pass a flow determined by the physical dimensions of the orifice.

Flow is then held back behind the low dam in a temporary reservoir which is slowly discharged through the orifice when the flood subsides.

Model orifice plate

An orifice plate is a device used for measuring the rate of fluid flow. It uses the same principle as a Venturi nozzle, namely Bernoulli’s principle which states that there is a relationship between the pressure of the fluid and the velocity of the fluid. When the velocity increases, the pressure decreases and vice versa.

An orifice plate is a thin plate with a hole in the middle. It is usually placed in a pipe in which fluid flows. When the fluid reaches the orifice plate, with the hole in the middle, the fluid is forced to converge to go through the small hole; the point of maximum convergence actually occurs shortly downstream of the physical orifice, at the so-called vena contracta point as presented in Figure 1. As it does so, the velocity and the pressure changes. Beyond the vena contracta, the fluid expands and the velocity and pressure change once again. By measuring the difference in fluid pressure between the normal pipe section and at the vena contracta, the volumetric and mass flow rates can be obtained from Bernoulli’s equation.

Principles of the Orifice: It is based on Bernoulli’s principle and is derived as follows. Bernoulli’s Equation for Incompressible Fluids for a parcel of fluid moving through a pipe with cross-sectional area “A”, the length of the parcel is d”x”, and the volume of the parcel A dx. If mass density is \( \rho \), the mass of the parcel is density multiplied by its volume \( \rho A \) dx. The change in pressure over distance dx is “dp” and flow velocity \( v = dx/dt \). Apply Newton’s Second Law of Motion we have:

\[
m \frac{du}{dt} = F
\]  
(1)

\[
\rho A \frac{du}{dt} = -Adp
\]  
(2)

\[
\rho \frac{du}{dt} = \frac{dp}{dx}
\]  
(3)
With density $\rho$ constant, the equation of motion can be written as:

$$
\frac{d}{dx} \left( \rho \frac{v^2}{2} + p \right) = 0
$$

by integrating equation (4) with respect to $x$.

$$
\frac{V^2}{2} + \frac{p}{\rho} = C
$$

Where, $C$ is a constant, sometimes referred to as the Bernoulli constant. It is not a universal constant, but rather a constant of a particular fluid system. Where, the speed is large, pressure is low and vice versa.

In the above derivation, no external work-energy principle is invoked. Rather, Bernoulli’s principle was
Putting equations (7) and (8) together, we have equation (9) or equation (10) 

\[
\Delta m \frac{p_1}{\rho} - \Delta m \frac{p_2}{\rho} + \Delta mgz_1 - \Delta mgz_2 = \frac{1}{2} \Delta m v_1^2 - \frac{1}{2} \Delta m v_2^2
\]

Alternatively

\[
\frac{1}{2} \Delta m v_1^2 + \Delta mgz_1 + \Delta \frac{p_1}{\rho}
= \frac{1}{2} \Delta m v_2^2 + \Delta mgz_2 + \Delta m \frac{p_2}{\rho}
\]

Dividing through equation (9 or 10) by \(\Delta m\) and \(v_1\) \(\Delta t\) the result is

\[
\frac{1}{2} v_1^2 + gz_1 + \frac{p_1}{\rho} = \frac{1}{2} v_2^2 + g(z_2 + p_2)\]

Equation (11) gives the basis for the modelling of the orifice plate.

The orifice plate condition: The method used is the free body flow diagram analysis of an orifice plate. Considering the above diagram with an assumption that system is in a steady state, incompressible, inviscid, laminar flow in a horizontal pipe (no change in elevation), with negligible frictional loss. Bernoulli’s equation reduces to an equation relating the conversion of energy between two points on the stream line as

\[
P_1 + \frac{1}{2} \rho \cdot V_1^2 = P_2 + \frac{1}{2} \rho \cdot V_2^2
\]
By continuity equation:

\[ Q = A_1 \cdot V_1 = A_2 \cdot V_2 \]  \hspace{1cm} (14)

Or

\[ V_1 = \frac{Q}{A_1} \]  \hspace{1cm} (15)

and

\[ V_2 = \frac{Q}{A_2} \]  \hspace{1cm} (16)

Substituting equations (15) and (16) into equation (13) gives

\[ P_1 - P_2 = \frac{1}{2} \cdot \rho \cdot \left( \frac{Q}{A_2} \right)^2 - \frac{1}{2} \cdot \rho \cdot \left( \frac{Q}{A_1} \right)^2 \]  \hspace{1cm} (17)

Making \( Q \) the subject of the formula from equation (17) we have

\[ Q = A_2 \sqrt{\frac{P_1 - P_2}{\rho}} \frac{1}{1 - (A_1 / A_2)} \]  \hspace{1cm} (18)

Equation (18) can be written as;

\[ Q = A_2 \sqrt{\frac{1}{1 - (d_2 / d_1)^4}} \sqrt{2 \left( P_1 - P_2 \right) / \rho} \]  \hspace{1cm} (19)

The above expression for \( Q \) gives the theoretical volume flow rate. Introducing the beta factor \( \beta = d_2 / d_1 \) as well as the coefficient of discharge we have

\[ Q = C_d A_2 \sqrt{\frac{1}{1 - \beta^4}} \sqrt{2 \left( P_1 - P_2 \right) / \rho} \]  \hspace{1cm} (20)

Since \( C_d \) ranges between 0.6 and 0.70 for orifice plate [3]

Finally, introducing the meter coefficient \( C \) which is defined as

\[ C = \frac{C_d}{\sqrt{1 - \beta^4}} \]

To obtain the final equation for the volumetric flow of the fluid through the orifice we have

\[ Q = C A_2 \sqrt{2 \left( P_1 - P_2 \right) / \rho} \]  \hspace{1cm} (21)

Equation (22) can be obtained by multiplying the density of the fluid by the mass flow rate at any section in the pipe

\[ \dot{m} = \rho \cdot Q = C A_2 \sqrt{2 \rho \left( P_1 - P_2 \right)} \]  \hspace{1cm} (22)

Pressure drop relationship; Making the pressure drop the subject of formula from equation (20) gives...
Table 2: Theoretical computation of functional parameters of orifice plate and gain

<table>
<thead>
<tr>
<th>$K_c$</th>
<th>$K_d$</th>
<th>$P-P_0$</th>
<th>$T_{(sec)}$</th>
<th>$\frac{1}{t}$ (sec $^{-1}$)</th>
<th>$P-P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>(\frac{7.8}{t} + 0.14)</td>
<td>1</td>
<td>1.00</td>
<td>7.94</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>(\frac{7.8}{t} + 0.34)</td>
<td>2</td>
<td>0.50</td>
<td>4.04</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>(\frac{7.8}{t} + 0.54)</td>
<td>3</td>
<td>0.33</td>
<td>3.14</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>(\frac{7.8}{t} + 0.74)</td>
<td>4</td>
<td>0.25</td>
<td>2.69</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>(\frac{7.8}{t} + 0.94)</td>
<td>4</td>
<td>0.20</td>
<td>2.50</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>(\frac{7.8}{t} + 1.14)</td>
<td>6</td>
<td>0.17</td>
<td>2.44</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>(\frac{7.8}{t} + 1.34)</td>
<td>7</td>
<td>0.14</td>
<td>2.45</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>(\frac{7.8}{t} + 1.54)</td>
<td>8</td>
<td>0.13</td>
<td>1.54</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>(\frac{7.8}{t} + 1.74)</td>
<td>9</td>
<td>0.11</td>
<td>2.61</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>(\frac{7.8}{t} + 1.94)</td>
<td>10</td>
<td>0.10</td>
<td>2.72</td>
</tr>
</tbody>
</table>

\[ \Delta P = P_1 - P_2 = \frac{Q^2 \rho (1 - \beta^4)}{2 C_d A_2^2} = \frac{Q^2 \rho (1 - \beta^4)}{2 C_d A_2^2 \beta^4} \]  
\[ (23) \]

**Density Relationship:** Making the density subject of formula from equation (20) gives

\[ \rho = \sqrt{(2 P_1 - P_2)} \cdot C_d A_2 \sqrt{(1 - \beta^4)} / Q \]  
\[ (24) \]

**Orifice Area Relationship:** Making the orifice area the subject of the formula from eqn 9 gives

\[ A_2 = \frac{Q \rho \sqrt{(2 P_1 - P_2)} \cdot C_d \sqrt{(1 - \beta^4)}}{Q} \]  
\[ (25) \]

**Viscosity relationship:** for flow characteristics we have

\[ Q = \pi \Delta PD^4 / 128 \mu L \]  
\[ (26) \]

**Computational procedure**

The following operational dimensions were arbitrarily chosen for data related to an orifice meter characteristics, diameter of the pipe = 240mm, diameter of the orifice 120mm, differential manometer reading = 400mmHg, discharge co-efficient = 0.65, density of the various fluid.
The data obtained were fed into the model computer using the developed mathematical expression in equations (19), (23), (24), (25) and (26).

Application of constant area, variable pressure drop meter as a proportional derivative control. The proportional derivative control works using the form stated below. Therefore by increasing output, with change in error.

\[ P = P_o + K_c + K_d \frac{dE}{dt} \quad (27) \]

Where, \( K_d \), gain, \( dE = \) change in error, \( dt = \) difference in time and \( P_o \) actual measurement.

This system is able to measure flow rate with pressure drop between the ranges of 400 to 850mm Hg. And it is found that after adjustment, the flow rate changes approximately 0.006 for 50mm Hg variation in pressure difference. And for such change, the error difference in flow rate is approximately 7.8 This means, at any given time change, this system will monitor flow rate using the following model

\[ P = P_o + K_c + K_d \frac{dE}{dt} \]

Fig. 7: Graph of Flow rate against change in Pressure for Engine oil

Fig. 8: Graph of change in control pressure (ΔP) of the orifice against proportional and derivative gain (\( k_c \) and \( k_d \))

\[ \rho_{H_2O} = 1000, \quad \rho_{\text{Petrol}} = 719.7, \quad \rho_{\text{Palm oil}} = 887, \quad \rho_{\text{Engine oil}} = 880\text{kg/m}^3 \text{ and pressure drop ranging from 450 to 850mmHg.} \]

For water at 400mmHg pressure drop: \( A_2 = \pi D^2/4 = (3.14*0.12^2)/4 = 0.0113 \text{m}^2 \), \( A_1 = 0.0452 \), \( \rho_w = 1000\text{kg/rn}^3 \), \( C_d = 0.65 \) and \( P_1 - P_2 = 400\text{mmHg} = 53328.96\text{Kg/m.s}^2 \)

\[ Q = A_2 \sqrt{\frac{2(P_1 - P_2)/\rho}{1 - (A_2 / A_1)^2}} \]

\[ Q = 0.0113*1.0327*10.954 = 0.1205\text{m}^3/\text{s} \]

But \( Q = \) theoretical

\[ Q_{\text{actual}} = Q_{\text{theoretical}} \times C_d = 0.1205*0.65 = 0.078 \approx 0.08 \]

Fig. 9: Graph of change in control pressure (ΔP) of the orifice against time (t)

Fig. 10: Graph of change in control pressure (ΔP) of the orifice against inverse of time (1/t)
The flow rate against change in pressure for engine oil is shown in Figure 7. From Figure 7, it is seen that increase in flow rate was observed with increase in change in pressure. The variation in flow rate can be attributed to the variation in change in pressure. The equation of the best fit is given as follows.
\[ Q = 11943 \times \Delta P - 612.26 \] and \( R^2 = 0.9988 \)
Whereas for theoretical \( Q_t = 768.65 \times \Delta P - 598.7 \) and \( R^2 = 0.9951 \).

Figure 8 illustrates change in control pressure of the orifice against proportional gain (\( K_p \)) and derivative gain (\( K_d \)). From Figure 8, it is seen that increase in the proportional/derivative gain (\( K_p, K_d \)) resulted to decrease in control pressure of the orifice. The variation in the control pressure can be attributed to the variable in the proportional/derivative gain (\( K_p, K_d \)). The equation of the best fit is given as:
\[ \Delta P = 316.83k_c^6 - 1199.8k_c^5 + 1837k_c^4 - 1458.8k_c^3 + 640.39k_c^2 - 150.8k_c + 17.899 \text{ and } R^2 = 0.9998 \]

Defining the polynomial curve of Figure 8, the effect of time on the control pressure of the orifice was illustrated in Figure 9. From Figure 9, increase in time resulted to decrease in control pressure of the orifice. The variation in the control pressure of the orifice can be attributed to the variation in time. The equation of the best fit for a parabolic curve is shown in Eq. 34, defining the polynomial curve of Figure 9. Figure 10 illustrate the relationship between control pressure of the orifice against inverse of time. From Figure 10 it is seen that increase in the inverse of time resulted to increased in the control pressure of the orifice. The variation in the control pressure of the orifice be attributed to the variation in the inverse of time (Eq. 35).

**RESULTS AND DISCUSSION**

Figure 4 illustrate the flow rate characteristics of theoretical and actual against change in pressure for water medium. It is seen that increase in pressure results to increase in flow rate. The equation of the best fit is given as \( Q_a = 13136 \times \Delta P - 659.67 \) with square root of \( R^2 \) = 0.9935 for actual whereas for theoretical \( Q_t = 8173 \times \Delta P - 597.69 \) and \( R^2 = 0.9974 \).

From Figure 5, the change in flow rate of petrol was examined with change in pressure. Results obtained revealed that increase in flow rate of petrol yielded increase in change of pressure. The equation of the best fit obtained is \( Q_a = 10550 \times \Delta P - 583.93 \) \( R = 0.9974 \) for actual and theoretical flow rate is \( Q_t = 6970 \times \Delta P - 604.52 \) and \( R^2 = 0.9969 \) as presented in Figure 5.

Results obtained in Figure 6 illustrate the flow rate characteristics of palm oil in an orifice plate. From Figure 6 increase in flow rate was observed with increase in change in pressure. The equation of the best fit is as follows
\[ Q_t = 11789 \times \Delta P - 592.78 \] \( R^2 = 0.996 \) for actual and for theoretical we have \( Q_t = 7758.5 \times \Delta P - 607.05 \) and \( R^2 = 0.9969 \)

**NOMENCLATURE**

- \( Q \) = volumetric flow rate (at any cross-section) \( (m^3/s) \)
- \( m \) = mass flow rate (at any cross-section), \( (kg/s) \)
- \( C_d \) = coefficient of discharge, dimensionless
- \( C \) = orifice flow coefficient, dimensionless
- \( A_1 \) = cross-sectional area of the pipe, \( (m^2) \)

**CONCLUSIONS**

The following conclusion was drawn from the findings. The effectiveness of orifice plate depends on the density, viscosity and other factor. Orifice plate mechanism in place for measuring flow is faster. The control pressure of the orifice is very important as one of the governing factors to reduce error in the proportional and derivative gain (\( K_p, K_d \)). As the proportional gain and derivative gain increases the control pressure of the orifice decrease. The results of the actual has a good match with theoretical showing how reliable the developed can be used in monitoring and predicting the flow characteristics of fluid flow in an orifice.
A_2 = cross-sectional area of the orifice hole, (m^2)
d_1 = diameter of the pipe, (m)
d_2 = diameter of the orifice hole, (m)
\( \beta \) = ratio of orifice hole diameter to pipe diameter, dimensionless
V_1 = upstream fluid velocity, (m/s)
V_2 = fluid velocity through the orifice hole, (m/s)
P_1 = fluid upstream pressure, Pa with dimensions of (kg/m s^2)
P_2 = fluid downstream pressure, Pa with dimensions of (kg/m^2)
\( \rho \) = fluid density, (kg/m^3)

REFERENCES


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