Modeling of the velocity profile of a bioreactor: the concept of biochemical process

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This research covers the modeling of the velocity profile of a bioreactor with recycle; the concept of biochemical process. The biochemical process adopted is fermentation and a plug-flow fermenter (PFF) was taken as a case study. The derivation of workable model equations for monitoring and predicting the velocity profile of a PFF were obtained, together with obtaining the model equations for investigating the effect of microbial and substrate concentrations on the discharge coefficient, bioreactor’s volume. Constant data were sourced from literatures, together with hypothetical values to simulate the derived model equations using Mathlab. The substrate concentration decreases with time as biomass population increases with time. Effect of biomass concentration on discharge coefficient, shows that increase in biomass concentration brings a corresponding increase in the discharge coefficient as well as the bioreactor’s volume revealed that substrate concentration is depleting alongside with bioreactor’s volume follows the same trend of change when substrate concentration is decreasing irrespective of whether the length or area of the bioreactor is varied. The effect of microbial concentration on bioreactor volume when area and length of bioreactor are varied reveals that the process followed same trend only that there is a presence of lag phase upon the influence of inhibitors. The inverse substrate concentration increases, the space velocity also increases, and there is a linear trend of change on the inverse substrate concentration with respect to space velocity.

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Capsule Summary: The velocity profile of a bioreactor based on biochemical parameters was modeled in present investigated.


INTRODUCTION

The velocity profile of a bioreactor is more or less known as the rate of reaction profile of a bioreactor. This defines the variation in the concentration of substrate or microbial with respect to time. In other words, it is how fast or slow a biochemical reaction takes place. The velocity (reaction rate) profile of a bioreactor is monitored in any manufactured or engineered device or system that supports a biologically
active environment. The rate of reaction occurs in a vessel in which a chemical process is carried out which involves organisms or biochemically active substances derived from such organisms. This process can either aerobic or anaerobic. For the purpose of this work we are taking a look at fermentation as a biochemical process taking place in a plug-flow Bioreactor (Fermenter) with recycle (Ukpaka, et al., 2009; Octave, 2007; William, 2007; Abashar and Butt, 200). Fermentation processes utilize microorganisms to convert solid or liquid substrates into various products. The substrates used vary widely, any material that supports microbial growth is a potential substrate. Similarly, fermentation-derived products show tremendous variety. Commonly consumed fermented products include bread, cheese, sausage, picked vegetables, beer, wine, citric acid, and soy sauce (Isla, et al., 1983., Jenning, 1991., Shah, 1969 & Copplestone).

In a bioreactor with recycle the effluent emerging from the reactor is fed into a settling unit. Microorganisms settle to the bottom of the tank, from where they are recycled into the reactor vessel. As a consequence of settling, the concentration of the microorganisms leaving the settling unit in the recycle stream is higher than that entering it from the biological reactor (singh and Saraf, 1981). The settling of the microorganisms greatly reduces their concentration in the effluent leaving the settling unit, producing a cleaner effluent stream. Recycle enables a higher concentration of microorganisms to be maintained in the bioreactor, which allows the reactor to run at much greater flow-rates and increase its efficiency (Ukpaka and Ogoni, 2015., Dyson and Simon, 1968; Hill, 1977; Elnashaie et al., 1988). The delimitation of Study includes: derivation of workable model equations to monitor the velocity profile of a plug-flow fermenter (PFF) with recycle and the use of MATLAB to simulate hypothetical values of a typical plug-flow fermenter using the derived workable model equations.

The velocity profile of a bioreactor is more or less known as the rate of reaction profile of a bioreactor. This defines the variation in the concentration of substrate or microbial with respect to time. In other words, it is how fast or slow a biochemical reaction takes place. This is measured by determining the change in the concentration with regard to time change. Enzymes are frequently used as catalysts to promote specific reactions in free solution. They are typically required in small amounts and are attractive in that they obviate both the need to provide the nutritional support which would be required for micro-organism to perform the same conversion, and the possible subsequent removal of those microbes. Furthermore, the enzyme need not necessarily be of microbial origin so that a wider choice of operating conditions and characteristics may be available.

The plug-flow tubular fermenter (PFTF) is in some respects the opposite limiting form of fermenter to the CSTF. In its idealized form, it is characterized by the fact that the liquid phase passes through the fermenter without backmixing. The fresh feed and the inoculum enter at one end of the fermenter and the mixture of the feed and the growing cells progress in unison towards the exit point. The small portion of the feed behaves as in a batch fermenter from its moment of entry up to the time where it leaves the fermenter, with the difference that the time co-ordinate of the batch fermentation is replaced by the time it takes to travel the axial distance along the fermenter tube. In practice, the fermenter may not necessarily be a tube and the idealized requirement of no back-mixing is never achieved.

**MATERIAL AND METHODS**

**Derivation of model equations**
Analysis of the operation of an idealized plug-flow fermenter (PFF) may be carried out by considering an elementary volume of the fermenter shown below.

The growth of biomass is initiated by the addition of an inoculum; in this case it is represented by the stream at a volumetric flow rate of \( F_0 \) with biomass concentration \( X_0 \) and substrate concentration \( S_0 \). As it exit the fermenter into the clarifier, a portion of it is recycled back into the fermenter and is mixed with the fresh feed as depicted above. It is assumed that there is no biochemical reaction or growth occurring in the clarifier, so that the substrate concentration \( S \) in the entering stream is the same as that in the clarified liquid effluent stream, in the recycle stream and in the exit biomass rich stream.

The general material balance statement is:

Material input + Formation by Biochemical reaction - Material output = Accumulation

\[ (1) \]

Material Balance in terms of Biomass (X)

Rate of reaction \( R=\)Velocity \( V \),

Where, input = \( (F_0 + F_R)X \), output = \( (F_0 + F_R)(X+dX) \),

Formation by biochemical reaction = \( RxdV \), and accumulation = \( \frac{dX}{dt} V \)

\[ (1a) \]

Considering when \( dV = acdz \), and at steady state equation \( (1a) \) becomes;

\( (F_0+F_R)X+Racdz-(F_0+F_R)(X+dX)=0 \) \quad \[ (1b) \]

Simplifying equation \( (1b) \) by substituting the above assumption we have

\( Racdz=F_0+F_R(dX) \) \quad \[ (1c) \]

Recycle ratio is given as \( R=\frac{F_R}{F_0} \) and by substituting the expression into equation \( (1c) \), therefore \( F_R=RF_0 \)

\( Racdz=F_0(1+R)dX \) \quad \[ (2) \]

Application of separating the variables into equation \( (2) \) and integrating, we have;

\[ \frac{ac}{F_0(1+R)} \int_0^Z dZ = \int_{X_A}^{X_E} \frac{dX}{RX} \] \quad \[ (3) \]

Integrating equation \( (3) \) at the boundary conditions of 0-Z and \( X_A-\)Xe, we have

\[ \frac{ac}{F_0(1+R)} \int_0^Z dZ = \int_{X_A}^{X_E} \frac{dX}{RX} \] \quad \[ (4) \]

Similarly, for the Substrate the form of the mathematical expression is given as:

\[ \frac{ac}{F_0(1+R)} \int_0^Z dZ = \int_{SA}^{Se} \frac{ds}{RS} \] \quad \[ (5) \]
If, \( S_A = \frac{S_0F_0}{(1+R)F_0+F_R} \)

\[ S_A = \frac{S_0F_0}{(1+R)F_0+F_R} \text{ and } S_A = \frac{S_0+R}{1+R} \]  
(6)

Similarly, for the biomass we have

\[ X_F = \frac{X_0Fo + X_RFR}{F_0+F_R} \]

If \( X_R = \frac{EXe}{Xe} \)

\[ X_A = \frac{X_0Fo + XeFoR}{F_0+F_R} \]

\[ X_A = \frac{X_0 + XeFR}{1+R} \]  
(7)

Substituting equations (6) and (7) into the boundary conditions on equations (5) and (4) as well as recalling equation (5) we have

\[ \frac{ac}{Fo(1 + R)} \int_0^Z dZ = \int_{SA}^{Se} dS \]

Recall that for Monod’s kinetics and Discharge Coefficient, the reaction rate with respect to the substrate \( Rs \) may be defined as;

\[ Rs = \frac{U_{mS}Xa}{(Ks+S)Y} \]

Substituting \( Rs \) into equation (5), we have

\[ \frac{ac}{Fo(1 + R)} \int_0^Z dZ = \int_{SA}^{Se} dS \]

Let \( S_A = S_i \). Integrating the equation (7a), we have

\[ \frac{acXaUmZ}{YFo(1 + R)} = Ksln \frac{Si}{Se} + (Se - Si) \]  
(8)

The definition of the Discharge Coefficient \( Y = \frac{Xe - Xi}{Si - Se} \) can be used to derive an expression for the Biomass concentration by substituting \( Y = \frac{Xe - Xi}{Si - Se} \) into equation (8) and expression obtained is given below,

\[ \frac{acXaUmZ}{Fo(1 + R)(Xe - Xi)} = Ksln \frac{Si}{Se} + (Si - Se)(Xe - Xi) \]

\[ \frac{acXaUmZ}{Fo(1 + R)} = (Xe - Xi)Ksln \frac{Si}{Se} + (Si - Se)(Xe - Xi) \]

\[ \frac{acXaUmZ}{Fo(1 + R)} = Xe - Xi Ksln \frac{Si}{Se} + (Xe - Xi) \]

\[ \frac{acXaUmZ}{Fo(1 + R)} = YKsln \frac{Si}{Se} + (Xe - Xi) \]  
(9)

Let us recall that for a given substrate concentration \( S \), the rate of biodegradation is expressed thus;

\[ \frac{ds}{dt} = -\beta S \]  
(9a)

On separation of variables for integration, the equation (9a) gives;
\[ \int_{S_i}^{S_e} \frac{ds}{S_e - S_i} = -\int_0^t \beta dt \]

Let \( t = \tau \)

\[ \ln \frac{S_i}{S_e} = \beta \tau \tag{10} \]

Substituting equation (10) into equation (8) gives;

\[ \frac{a c X a U m Z}{Y F o (1 + R)} = K s \beta \tau + (S_i - S_e) \]

Assuming \( X_a = X_i \)

\[ S_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} + S_i \right) + (K_s \beta) \tau \tag{11} \]

Equation (11) is the Model equation for predicting the velocity (reaction rate) profile in terms of substrate concentration of a plug-flow fermenter.

Similarly, substituting equation (10) into equation (9) gives;

\[ \frac{a c X a U m Z}{Y F o (1 + R)} = Y K_s \beta + (X_e - X_i) \]

Assuming \( X_a = X_i \)

\[ X_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} + X_i \right) - (Y K_s \beta) \tau \tag{12} \]

Equation (12) is the Model equation for predicting the velocity (reaction rate) profile in terms of biomass concentration of a plug-flow fermenter.

**RESULTS AND DISCUSSION**

For Fig 5 shown, which is the graph of effluent substrate concentration \( S_e \) and effluent microbial concentration \( X_e \) against space time \( \tau \), the following equations were used to simulate the parameters;

\[ S_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} + S_i \right) + (K_s \beta) \tau \]

and \( X_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} + X_i \right) - (Y K_s \beta) \tau \)

The following parameters were chosen in simulating the developed model using matlab computer programme language, such as: \( F_o = 18L/hr \), \( U_m = 0.25/hr \), \( K_s = 0.12g/hr \), \( Y = 0.42 \), \( R = 1 \), \( Z = 10m \), \( a_c = 0.5m^2 \).

\( T: 0, 5, 10, 15, 20, 25, 30 \), Influent Substrate: 2000, 1900, 1800, 1700, 1650, 1600, 1500, Influent Microbial: 2000, 1300, 6800, 9000, 13000, 18000, 1800, bioreactor’s area: 0.1, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, Bioreactor's length: 2, 5, 7, 8, 10, 15, 20 and \( \beta = 1/hr \)

From the graph of Figure 2, it can be seen that the concentration of substrate reduces with time which implies that as the microbial begins to feed on the substrate, the substrate begins to reduce. Considering the curve of microbial concentration, one will notice that there is a slight decline in microbial concentration initially which is likened to as lag phase. In this lag phase adaptation is taking place by the microbes. This is because when the microbes are introduced some may initially launch attack on the substrate without observing the environment and therefore result to the death of microbes, while others will gradually observe the environment before launching attack on the substrate. After the lag phase comes the acceleration phase which depicts slight increase in microbial population before the progressive phase showing non-slight increase in microbial population. A stationary phase is seen from the graph showing that the microbes growth are stagnant.

Fig 3 is showing the graph of effluent microbial \( X_e \) against discharge coefficient \( Y \), the following data were used for simulation on this model equation;

\[ X_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} + X_i \right) - (K_s \beta) Y \]

The above graph shown in Figure 3 reveals that as the microbial population increases the discharge coefficient which defines the yield also increases. This shows that increase in microbial population have a positive effect on the yield.

Figure 4 is showing a graph of effluent substrate concentration \( S_e \) against bioreactor’s volume \( a \) at varying bioreactor’s cross sectional area \( a_c \); these data were used for simulating this equation; \( S_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} \right) a_c + (S_i + K_s \beta) a_c \).

Graph shown in Figure 5 reveals that decrease in substrate concentration leads to increase in bioreactor’s volume. The curve justifies the fact that the smaller the concentration of substrate in a vessel the more the vessel appears to be large for such substrate, and if the substrate is increased the volume of the vessel will appear small for the substrate. For figure 5 showing the graph of effluent substrate concentration \( S_e \) against bioreactor’s volume \( a \) at varying bioreactor’s length, the following data were used for simulating this model equation; \( S_e = \left( \frac{a c X i U m Z}{Y F o (1 + R)} \right) a_c + (S_i + K_s \beta) a_c \).

For the above graph presented in Figure 6, there is same effect of effluent substrate concentration on bioreactor’s volume at varying bioreactors length as well as varying bioreactor’s area.

Figure 7 is showing effluent microbial concentration \( X_e \) against bioreactor’s volume \( a \) at varying bioreactor’s cross sectional area \( a_c \) the following data were used to simulate this equation;

\[ X_e = (X_i - Y K_s \beta) T + \left( \frac{a c X i}{Y F o (1 + R)} \right) a_c \]

For Figure 8 showing effluent microbial concentration \( X_e \) against bioreactor’s volume \( a \) at varying bioreactor’s length \( Z \), the following data were used to simulate this equation;

\[ X_e = (X_i - Y K_s \beta) T + \left( \frac{a c X i}{Y F o (1 + R)} \right) a_c \]
This graph also show the same effect just as Fig 7, only that initially there is a lag phase present for this curve.

Figure 9 is showing the graph of inverse effluent substrate concentration against space velocity, the following tabulated data were used to simulate this equation;

\[
\frac{1}{Se} = \left( -\frac{YFo(1+R)}{acZXiUm} + S_e \frac{1}{Se} \right) + (Ks\beta)^\frac{1}{T}
\]

Figure 9 shows that as the inverse of the substrate concentration increases, the space velocity also increases. There is a linear trend of change in the inverse substrate concentration against space time.
CONCLUSIONS

The following conclusions were drawn from this research work. Matlab is a good mathematical tool for simulating a process such as bioreactor with sludge recycles. The MathLab program was used for monitoring and predicting the rate of change in concentration at different space time. The relationship between the concentration and discharge coefficient was monitored using MathLab program. The relationship between the concentration and bioreactor volume was monitored using MathLab program. The mathematical approach of modeling the dynamics of a bioreactor with sludge recycle can be applied to other fields of engineering.

Nomenclature

\( a_c = \) Cross Sectional Area of a Bioreactor
\( F_0 = \) Volumetric Flowrate of Feed
\( K_s = \) Monod's Reaction Rate Constant
\( R = \) Recycle Fraction
\( U_m = \) Maximum Specific Growth Rate
\( T = \) Space Time
\( X_i = \) Influent Microbial Concentration
\( X_e = \) Effluent Microbial Concentration
\( X_r = \) Recycled Microbial Concentration
\( S_i = \) Influent Substrate Concentration
\( S_e = \) Effluent Substrate Concentration
\( \beta = \) Reaction Rate Constant
\( Y = \) Discharge Coefficient
\( Z = \) Bioreactor's Length

REFERENCES


Appendix (supplementary material)

Mathlab program

% Modelling of the velocity profile of a bioreactor with Recycle:

Fo=18;%l/hr
Um=0.25;%l/hr
Ks=0.12;
Y=0.42;
R=1;
Z=10;
Ac=0.5;%m2
B=1;%l/hr
for Si=[2000,1900,1800,1700,1650,1600,1500];
Xi=[2000,1300,6800,9000,13000,18000,18000];
V=[0.5,10,15,20,25,30];%SpaceTime
Se=(Si-((Ac*X*I*Um)/(Y*Fo*(1+R))))+(Ks*B).*V;
end
formatrat
disp(Se)

OUTPUT:
Columns 1 through 6
42709/32  121202/87  30988/33  79564/105
67571/158  313/21
Column 7
1628/105

Fo=18;%l/hr
Um=0.25;%1/hr
Ks=0.12;
Y=0.42;
R=1;
Z=10;
Ac=0.5;%m2
B=1;%1/hr
for Si=[100,100,80,50,15,5,2];
Xi=[2000,1300,6800,9000,13000,18000,18000];
V=[0,5,10,15,20,25,30];
Xe=(Xi+(Ac*Z*Xi*Um)/(Fo*(1+R)))-(Ks*B).*V;
end
formatrat
disp(Xe)
OUTPUT:
Columns 1 through 6
18625/9  51113/38  56287/8  232799/25
94153/7  37247/2
Column 7
93116/5

Y=[0.1,0.1,0.2,0.3,0.4,0.5,0.5];
Xe=[18625/9,51113/38,56287/8,232799/25,94153/7,37247/2]
plot(Y,Xe)
xlabel('Discharge Coefficient')
ylabel('Effluent microbial Conc')
title('A GRAPH OF EFFLUENT MICROBIAL CONCENTRATION AGAINST DISCHARGE COEFFICIENT')

formatrat
disp(Se)
OUTPUT:
Columns 1 through 6
42541/29  28103/19  3829/3  36938/35
11567/18  313/21

Se=[
  42709/32,121202/87,30988/33,79564/105,67571/158,
  313/21,1628/105 ]
Xe=[18625/9,51113/38,56287/8,232799/25,94153/7,37247/2]
V=[0,5,10,15,20,25,30];
plot(V,Se,V,Xe)
xlabel('Space Time')
ylabel('Effluent Substrate Concentration')
title('A GRAPH OF EFFULENT SUBSTRATE CONC AND EFFULENT MICROBIAL CONC AGAINST SPACE TIME')
legend('EffSubtConcent','Eff Micro Conc')

Fo=18;%l/hr
Um=0.25;%1/hr
Ks=0.12;
R=1;
Z=10;
Ac=0.5;%m2
B=1;%1/hr
for Si=[100,100,80,50,15,5,2];
Xi=[2000,1300,6800,9000,13000,18000,18000];
V=[0,5,10,15,20,25,30];
Y=[0.1,0.1,0.2,0.3,0.4,0.5,0.5];
r=1/Ks*B;
Xe=(Xi+(Ac*Z*Xi*Um)/(Fo*(1+R)))-(Ks*B*V).*Y;
end
formatrat
disp(Se)
OUTPUT:
Columns 1 through 6
42541/29  28103/19  3829/3  36938/35
11567/18  313/21

Column 7
-9874/35

%Bio Reactor Vol
Z=10;
Ac=[0.1,0.1,0.2,0.3,0.4,0.5,0.6];

biovol=Z.*Ac;
disp(biovol)

OUTPUT:
Columns 1 through 6
1 1 2 3 4 5
Column 7
6

-------------------------
%A GRAPH OF EFFLUENT SUBSTRATE CONCENTRATE SE AGAINST BIOREACTOR VOLUME
BioVol=[ 1,1,2,3,4,5,6 ];
Se=[42541/29,28103/19,36938/3,36938/35,11567/18,9874/35 ]
plot(BioVol,Se)
xlabel('BIOREACTOR VOLUME(Ac.Z)')
ylabel('EFFLUENT SUBSTRATE CONC(Se)')
title('A GRAPH OF EFFLUENT SUBSTRATE CONCENTRATE SE AGAINST BIOREACTOR VOLUME(VARIED AREA)')

%Bio Reactor Vol GRAPH 7
Ac=0.5;
Z=[2,5,7,8,10,15,20];
biolv=Ac.*Z;
disp(biovol)

OUTPUT:
Columns 1 through 6
1 5/2 7/2 4 5 15/2
Column 7
10
BioRVol=[ 1.5/2,7/2,4,5,15/2,10 ];
Se=[42541/29,28103/19,36938/3,36938/35,11567/18,9874/35 ];
plot(BioVol,Se)
xlabel('BIOREACTOR VOLUME(Ac.Z)')
ylabel('EFFLUENT SUBSTRATE CONC(Se)')
title('A GRAPH OF EFFLUENT SUBSTRATE CONCENTRATE SE AGAINST BIOREACTOR VOLUME(VARIED LENGTH)')

OUTPUT:
Columns 1 through 6
1 5/2 7/2 4 5 15/2
Column 7
10
BioRVol=[ 1.5/2,7/2,4,5,15/2,10 ];
Se=[42541/29,28103/19,36938/3,36938/35,11567/18,9874/35 ];
plot(BioVol,Se)
xlabel('BIOREACTOR VOLUME(Ac.Z)')
ylabel('EFFLUENT SUBSTRATE CONC(Se)')
title('A GRAPH OF EFFLUENT SUBSTRATE CONCENTRATE SE AGAINST BIOREACTOR VOLUME(VARIED LENGTH)')
\[
V = [0.5, 10, 15, 20, 25, 30];
Xe = (Xi - Y*Ks*B.*V) + (Xi*Um)/Fo*(1+R).*Ac*Z;
\]

end
formatrat
disp(Xe)

Ac=0.5;

Z=[2,5,7,8,10,15,20]
ReacVol=Ac.*Z;
disp(ReacVol)

Vol=[1.5,2,7/2,4,5.15,2,10];
Xe=[18500/9,94846/71,178950/11,38997/4,101104/7,81995/4,41997/2];
plot(Vol,Xe)
xlabel('BIOREACTOR VOLUME(Ac.Z)')
ylabel('EFFLUENT MICROBIAL CONC(Xe)')
title('A GRAPH OF EFFL MICROBIAL CONCENTRATE Xe AGAINST BIOREACTOR VOLUME(VARIED LENGTH)')

Y=0.42;
R=1;
Z=10;
Ac=0.5;%m2
B=1;%1/hr
for Si=[2000,1900,1800,1700,1650,1600,1500];
Xi=[2000,1300,6800,9000,13000,18000,18000];
V=0:5:30;
p=1./Si;
q=(Y*Fo*(1+R));
r=(Ac*Z.*Xi*Um);
s=1/(Ks*B);
t=1./V;
InSe=(p+(q./r))+s.*t;
end
formatrat
disp(InSe)
OUTPUT:
Columns 1 through 6
1/0 3583/2161 873/1049 91/164 1447/3475 20833/62500
Column 7
2893/10415
\[
inV=[1/0,1/5,1/10,1/15,1/20,1/25,1/30];
inXe=[-1/0,1-732/439,2839/1333,4337/3287,-2032/2053,-1036/1649,-2121/3215];
plot(inV,inXe)
xlabel('Inverse Space Velocity')
ylabel('Inverse Effluent Substrate Conc')
title('A GRAPH OF Effluent Substrate Conc Against Space Time[Inverse Value]')
inSe=[
1/0,3583/2161,873/1049,91/164,1447/3475,20833/62500,10415];
inV=[1/0,1/5,1/10,1/15,1/20,1/25,1/30];

%inXe=[-1/0,1-732/439,2839/1333,4337/3287,-2032/2053,-1036/1649,-2121/3215];
plot(inV,inSe,inV,inXe)
xlabel('Space Velocity')
ylabel('Inverse Effluent Substrate Conc')
title('A GRAPH OF Effluent Substrate Conc Against Space Time[Inverse Value]')